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"THE FIELD OF A CHARGED POINT MASS"

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[Note: the following report appeared in the regular "Letters to the Editor" section of the monthly Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki, Volume 22, No 1 (January 1952), page 125.]

In the theory of gravitation, it is demonstrated that the energy of matter and of a field, which energy is included in a volume bounded by a closed surface, can be completely determined by the values of the gravitational potentials and of their derivatives on this surface, and can be expressed by the following surface integral [1]:

$$E = (1/2\kappa) \; (\partial/\partial x^i) \left[\sqrt{g} \, (g^{44}g^{ik} - g^{4i}g^{4k}) \right] df_k, \quad (1)$$

$(i,k = 1,2,3)$.

The problem concerning the field of a charged point mass permits the exact joint solution of the equations of gravitation and electrodynamics, and gives the following expression for the gravitational potential [2,3]:

$$g^{ik} = \delta^{ik} - \left(\frac{\gamma}{r} - \frac{\varepsilon^2}{4r^2}\right)\frac{x^i x^k}{r^2}; \quad g^{ik} = 0, \quad g^{44} = \left(1 - \frac{\gamma}{r} + \frac{\varepsilon^2}{4r^2}\right)^{-1}, \quad (2)$$

where the radii ε and γ are connected with the charge e and mass m of the source by the following formulas:

$$\varepsilon = e\sqrt{\frac{\kappa}{2\pi}}, \quad \gamma = \frac{mc^2\kappa}{4\pi} \quad (3)$$

Set (2) into (1) and, integrating over the surface of a sphere with infinitely great radius, we obtain:

$$E = mc^2 \quad (4)$$

This result could have been foreseen, in as much as the electrical terms in (2) fall off at infinity faster than the gravitational terms.

In as much as the energy of the electrical field excited by a charge drops out from expression (4), we can conclude that solution (2) leads to a contradiction and does not possess physical significance, although it is an exact solution of the equations of gravitation and electrodynamics.

In 5-optics /five-dimensional description/ it is shown that a charge, besides the electrical and gravitational field, excites also a scalar chi-field, which field is ignored in present-day physics and possesses a notable magnitude only at distances of the order of the radius ε from the charge. Instead of (2), 5-optics /five-dimensional description/ gives the following expression /4/: $g^{ik} = \delta^{ik} - \gamma x^i x^k \cdot r^{-3}$; $g^{i4} = 0$;

$$g^{44} = \left(1+\frac{\alpha}{r}\right)\Big/\left(1-\frac{\gamma}{r}\right); \quad \chi = \alpha/r, \tag{2'}$$

where $\alpha = \frac{1}{2}\left[\sqrt{\gamma^2+4\varepsilon^2} - \gamma\right]$.

If we take the chi-field into consideration, then integral (1) assumes the following form:

$$E = \frac{1}{2\kappa}\int \frac{\partial}{\partial x^i}\left[g(1+\chi)(g^{44}g^{ik} - g^{i4}g^{k4})\right] df_k . \tag{1'}$$

Substituting (2') and (1") and integrating over a sphere of infinitely large radius, we find instead of (4) the following:

$$E = \frac{3}{4}mc^2 + \sqrt{\frac{1}{16}(mc^2)^2 + 2\pi e^2 \kappa^{-1}} . \tag{4'}$$

For the energy of a point mass ($e \to 0$), we obtain $E_\gamma = mc^2$. For the energy of a point charge ($m \to 0$), we obtain:

$$E_\varepsilon = e\sqrt{\frac{2\pi}{\kappa}} = \frac{e^2}{\varepsilon} . \tag{5}$$

We see that in 5-optics /five-dimensional description/ the energy of the field of a point charge is obtained as finite. In the limit transition (passage) to the classical theory, $\kappa \to 0$, the energy becomes infinite.

Submitted 1 July 1951.

LITERATURE

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[3] W. Pauli. Theory of Relativity, Section 59 [translated into Russian, GTTI : 1947].

[4] Yu. Rumer. Ibidem (Jhurnal Eksper i Teoret Fiziki), Volume 19 (1949), 3.

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